

University of Groningen

Gauged supergravities from Bianchi's group manifolds

Bergshoeff, E; Gran, U; Linares, R; Nielsen, M; Ortin, T; Roest, D

Published in:
Classical and Quantum Gravity

DOI:
[10.1088/0264-9381/21/10/022](https://doi.org/10.1088/0264-9381/21/10/022)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2004

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Bergshoeff, E., Gran, U., Linares, R., Nielsen, M., Ortin, T., & Roest, D. (2004). Gauged supergravities from Bianchi's group manifolds. *Classical and Quantum Gravity*, 21(10), S1501-S1507. [PII S0264-9381(04)76676-9]. <https://doi.org/10.1088/0264-9381/21/10/022>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Gauged supergravities from Bianchi's group manifolds

Eric Bergshoeff¹, Ulf Gran², Román Linares³, Mikkel Nielsen¹,
Tomás Ortín⁴ and Diederik Roest¹

¹ Centre for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

² Department of Mathematics, King's College London, Strand, London WC2R 2LS, UK

³ Departamento de Física, Universidad Autónoma Metropolitana—Iztapalapa, AP 55-534, México, DF, cp 09340, Mexico

⁴ Instituto de Física Teórica, C-XVI, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

E-mail: d.roest@phys.rug.nl

Received 24 February 2004

Published 20 April 2004

Online at stacks.iop.org/CQG/21/S1501 (DOI: 10.1088/0264-9381/21/10/022)

Abstract

We construct maximal $D = 8$ gauged supergravities by the reduction of $D = 11$ supergravity over three-dimensional group manifolds. Such manifolds are classified into two classes, A and B, and eleven types. This Bianchi classification carries over to the gauged supergravities. The class A theories have 1/2 BPS domain wall solutions that uplift to purely gravitational solutions consisting of 7D Minkowski and a 4D Euclidean geometry. These geometries are generically singular. The two regular exceptions correspond to the near-horizon limit of the single- or double-centre Kaluza–Klein monopole. In contrast, the class B supergravities are defined by a set of equations of motion that cannot be integrated to an action and allow for no 1/2 BPS domain walls.

PACS numbers: 04.50.+h, 04.65.+e, 11.25.-w

1. Introduction

The first example of a maximal $D = 8$ gauged supergravity is the $SO(3)$ gauged supergravity constructed in 1985 by Salam and Sezgin [1]. This theory occurs in the DW/QFT correspondence when one considers the near-horizon limit of the D6-brane or the Kaluza–Klein monopole [2]. Also, a number of papers appeared where it played an important role in the construction of special holonomy manifolds by considering wrapped branes (see, e.g., [3–6]).

In view of these and more applications, it is of interest to ask oneself how unique the $SO(3)$ gauged supergravity is. In the recent papers [7, 8], we performed a generalized dimensional reduction of $D = 11$ supergravity over three-dimensional group manifolds [9, 10], leading to

Table 1. The Bianchi classification of the different 3D Lie algebras in terms of components of their structure constants. There is one additional special case: type III = type VI_a with $a = \frac{1}{2}$.

Class A	a	(q_1, q_2, q_3)	Algebra	Class B	a	(q_1, q_2, q_3)	Algebra
IX	0	(1, 1, 1)	$so(3)$				
VIII	0	(1, -1, 1)	$so(2, 1)$				
VII ₀	0	(0, 1, 1)	$iso(2)$	VII _a	a	(0, 1, 1)	$iso(2)_a$
VI ₀	0	(0, -1, 1)	$iso(1, 1)$	VI _a	a	(0, -1, 1)	$iso(1, 1)_a$
II	0	(0, 0, 1)	$Heis$	IV	1	(0, 0, 1)	$Heis_{a=1}$
I	0	(0, 0, 0)	$u(1)^3$	V	1	(0, 0, 0)	$u(1)^3_{a=1}$

more general gauge groups in 8D. Such manifolds are labelled by their algebra of isometries that act transitively on the manifold.

For three-dimensional Lie groups, the structure constants have nine components, which can be conveniently parametrized by

$$f_{mn}^p = \epsilon_{mnq} Q^{pq} + 2\delta_{[m}^p a_{n]}, \quad Q^{pq} a_q = 0, \quad (1)$$

where $Q^{pq} = \frac{1}{2} \text{diag}(q_1, q_2, q_3)$ and $a_m = (a, 0, 0)$. The constraint on their product follows from the Jacobi identity. The different possibilities are organized in the Bianchi classification [11]. One distinguishes between algebras of class A with $f_{mn}^n = 0$ and class B with $f_{mn}^n \neq 0$. For the class B algebras we will use a notation that indicates that each class B algebra can be viewed as a deformation (with deformation parameter a) of a class A algebra (see table 1).

By using class A group manifolds we have constructed six maximal $D = 8$ supergravities [7] with gauge groups $SO(3)$, $SO(2, 1)$, $ISO(2)$, $ISO(1, 1)$, $Heis$ and $U(1)^3$. Here $Heis$ denotes the three-dimensional Heisenberg group (with generators corresponding to position, momentum and identity). The theory with gauge group $U(1)^3$ is obtained from a reduction over a torus T^3 and is referred to as the ungauged theory, since there are no fields that carry any of the $U(1)$ charges. All groups mentioned above are related to $SO(3)$ by group contraction and/or analytic continuation. We will refer to them as class A supergravities.

By using class B group manifolds yet more gauged supergravities can be constructed [8], whose gauge groups can be seen as deformations of $ISO(2)$, $ISO(1, 1)$, $Heis$ and $U(1)^3$ with deformation parameter a , as indicated in table 1. We call these class B supergravities. There is an extensive literature on the fact that a class B group manifold reduction leads to inconsistent field equations when reducing the action (for a recent overview see [12]). This is related to the fact that the field equations following from the reduced action do not coincide with the reduction of the field equations themselves [13]. We have found, by explicitly performing the group manifold procedure, that the reduction can be performed on-shell, i.e. at the level of the equations of motion or the supersymmetry transformations. Particularly, in string theory this seems to be a relevant approach since the worldsheet theory yields spacetime field equations rather than an action principle. We note that, although the $D = 8$ class B supergravities have no Lagrangian, there is a hidden Lagrangian in the sense that these theories can be obtained by dimensional reduction of a theory in $D = 11$ dimensions with a Lagrangian.

Note that among the different group manifolds there are a number of non-compact manifolds, in particular all class B group manifolds. Thus, many of the reductions we perform are not compactifications in the usual sense (on a small internal manifold) but rather consistent truncations of the full higher-dimensional theory to a lower-dimensional subsector. The consistency of the truncation guarantees that $D = 8$ solutions uplift to $D = 11$ solutions. We do not consider global issues here and focus on local properties.

We will first outline the group manifold reduction to 8D gauged supergravities. After constructing the different theories, we investigate the 1/2 BPS domain wall solutions for both classes A and B. Also, the uplifting of the different domain wall solutions to M-theory and the relations to the Kaluza–Klein monopole are discussed.

2. Reduction over a 3D group manifold

The reduction ansatz is formally the same for classes A and B and it involves the following fields,

$$\begin{aligned} 11\text{D} : \quad & \{\hat{e}_{\hat{\mu}}^{\hat{a}}, \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{\psi}_{\hat{\mu}}\}, \\ 8\text{D} : \quad & \{e_{\mu}^a, L_m^i, \varphi, \ell, A_{\mu}^m, V_{\mu mn}, B_{\mu\nu m}, C_{\mu\nu\rho}, \psi_{\mu}, \lambda_i\}, \end{aligned} \quad (2)$$

where the indices are defined according to an 8 + 3 split of the 11-dimensional spacetime: $x^{\hat{\mu}} = (x^{\mu}, z^m)$ with $\mu = (0, 1, \dots, 7)$ and $m = (1, 2, 3)$. Spacetime indices are written as $\hat{\mu} = (\mu, m)$, while the tangent indices are $\hat{a} = (a, i)$. The three-dimensional space is taken to be a group manifold and we reduce over its three (non-Abelian) isometries.

Using a particular Lorentz frame, the reduction ansatz for the 11-dimensional bosonic fields is

$$\hat{e}_{\hat{\mu}}^{\hat{a}} = \begin{pmatrix} e^{-\frac{1}{6}\varphi} e_{\mu}^a & e^{\frac{1}{3}\varphi} L_m^i A_{\mu}^m \\ 0 & e^{\frac{1}{3}\varphi} L_n^i U_n^m \end{pmatrix} \quad (3)$$

and

$$\begin{aligned} \hat{C}_{abc} &= e^{\frac{1}{2}\varphi} C_{abc}, & \hat{C}_{abi} &= L_i^m B_{abm}, \\ \hat{C}_{aij} &= e^{-\frac{1}{2}\varphi} L_i^m L_j^n V_{amn}, & \hat{C}_{ijk} &= e^{-\varphi} \epsilon_{ijk} \ell. \end{aligned} \quad (4)$$

The reduction ansatz for the fermions, including the supersymmetry parameter $\hat{\epsilon}$, reads as follows:

$$\hat{\psi}_{\hat{a}} = e^{\varphi/12} (\psi_a - \frac{1}{6} \Gamma_a \Gamma^i \lambda_i), \quad \hat{\psi}_i = e^{\varphi/12} \lambda_i, \quad \hat{\epsilon} = e^{-\varphi/12} \epsilon. \quad (5)$$

The matrix L_m^i describes the five-dimensional $SL(3, \mathbb{R})/SO(3)$ scalar coset space. It transforms under a global $SL(3, \mathbb{R})$ acting from the left and a local $SO(3)$ symmetry acting from the right. We take the following explicit representative, thus gauge fixing the local $SO(3)$ symmetry:

$$L_m^i = \begin{pmatrix} e^{-\sigma/\sqrt{3}} & e^{-\phi/2+\sigma/2\sqrt{3}} \chi_1 & e^{\phi/2+\sigma/2\sqrt{3}} \chi_2 \\ 0 & e^{-\phi/2+\sigma/2\sqrt{3}} & e^{\phi/2+\sigma/2\sqrt{3}} \chi_3 \\ 0 & 0 & e^{\phi/2+\sigma/2\sqrt{3}} \end{pmatrix}, \quad (6)$$

which contains two dilatons, ϕ and σ , and three axions χ_m . It is convenient to define the local $SO(3)$ invariant scalar matrix

$$\mathcal{M}_{mn} = -L_m^i L_n^j \eta_{ij}, \quad (7)$$

where $\eta_{ij} = -\mathbb{I}_3$ is the internal flat metric.

The only internal coordinate dependence in the ansatz appears via the matrix U_n^m , which is defined in terms of the left-invariant Maurer–Cartan 1-forms of a three-dimensional Lie group

$$\sigma^m \equiv U_n^m dz^n. \quad (8)$$

By definition, these 1-forms satisfy the Maurer–Cartan equations

$$d\sigma^m = -\frac{1}{2} f_{np}^m \sigma^n \wedge \sigma^p, \quad f_{mn}^p = -2(U^{-1})^r_m (U^{-1})^s_n \partial_{[r} U^p_{s]}, \quad (9)$$

where the $f_{mn}{}^p$ are independent of z^m and form the structure constants of the group manifold. Note that we use a slight extension of the original procedure of Scherk and Schwarz [10] by allowing for structure constants with non-vanishing trace (leading to class B supergravities). We find, by explicitly performing the group manifold procedure, that the class B reduction can be performed on-shell, i.e. at the level of the equations of motion or the supersymmetry variations, but not at the level of the action. Indeed, the lower-dimensional field equations cannot be integrated to an action.

An explicit representation of the Maurer–Cartan 1-forms for the general rank of the matrix Q was given in [7] in the case of class A. Including class B, i.e. $a \neq 0$, leads to the following matrix,

$$U^m{}_n = \begin{pmatrix} 1 & 0 & -s_{1,3,2} \\ 0 & e^{az^1} c_{2,3,1} & e^{az^1} c_{1,3,2} s_{2,3,1} \\ 0 & -e^{az^1} s_{3,2,1} & e^{az^1} c_{1,3,2} c_{2,3,1} \end{pmatrix}, \quad (10)$$

where we have used the following abbreviations,

$$c_{m,n,p} \equiv \cos\left(\frac{1}{2}\sqrt{q_m}\sqrt{q_n}z^p\right), \quad s_{m,n,p} \equiv \sqrt{q_m} \sin\left(\frac{1}{2}\sqrt{q_m}\sqrt{q_n}z^p\right)/\sqrt{q_n}, \quad (11)$$

and it is understood that the structure constants satisfy the Jacobi identity, amounting to $q_1 a = 0$.

With the ansatz above, class B gauged supergravities can be obtained. For our present purposes, it is enough to reduce the supersymmetry transformation rules. Since we are primarily interested in domain wall solutions, we will truncate the reduction to just include the following fields: $g_{\mu\nu}$, $L_m{}^i$ and φ . The resulting $D = 8$ fermionic transformations are

$$\begin{aligned} \delta\psi_\mu &= 2\partial_\mu\epsilon - \frac{1}{2}\Gamma^{ab}\omega_{\mu ab}\epsilon + \frac{1}{2}Q_{\mu ij}\Gamma^{ij}\epsilon + \frac{1}{24}e^{-\varphi/2}f_{ijk}\Gamma^{ijk}\Gamma_\mu\epsilon - \frac{1}{6}e^{-\varphi/2}f_{ij}{}^j\Gamma_\mu\Gamma^i\epsilon, \\ \delta\lambda_i &= -P_{\mu ij}\Gamma^\mu\Gamma^j\epsilon - \frac{1}{3}\Gamma^\mu\partial_\mu\varphi\Gamma_i\epsilon - \frac{1}{4}e^{-\varphi/2}(2f_{ijk} - f_{jki})\Gamma^{jk}\epsilon, \end{aligned} \quad (12)$$

where we have used the abbreviations $f_{ijk} \equiv L_i{}^m L_j{}^n L_{pk} f_{mn}{}^p$ and

$$P_{\mu ij} + Q_{\mu ij} \equiv L_i{}^m \mathcal{D}_\mu L_{mj}, \quad (13)$$

where P is symmetric and traceless and Q is antisymmetric. The full supersymmetry rules, without truncation, can be found in [7, 8].

The global duality group $GL(3, \mathbb{R})$ acts on the indices m, n, p in the obvious way and its action is explicitly given in [7]. In the gauged theory this, in general, is no longer a symmetry since it does not preserve the structure constants. The unbroken part is exactly given by the automorphism group of the structure constants as given in [7]. Of course it always includes the gauge group, which is embedded in $GL(3, \mathbb{R})$ via

$$g_n{}^m = e^{\lambda^k f_{kn}{}^m}, \quad (14)$$

where λ^k are the local parameters of the gauge transformations. However, the full automorphism group can be bigger; for instance it is nine-dimensional in the $U(1)^3$ case. Of course this amounts to the fact that the ungauged $D = 8$ theory has a $GL(3, \mathbb{R})$ symmetry. All other cases have $\text{Dim}(\text{Aut}) < 9$ and thus break the $GL(3, \mathbb{R})$ symmetry to some extent. For instance, the $SO(1, 1)$ subgroup corresponding to the determinant of the $GL(3, \mathbb{R})$ element is broken by all non-vanishing structure constants.

The higher-dimensional origin of the different gaugings can be viewed in another way. Note that the reduction ansatz (10) from 11D is independent of z^3 for all cases. This means that the reduction over 3D group manifolds can be split up: one first performs a trivial reduction over z^3 from 11D to 10D, after which one is left with a non-trivial reduction ansatz from 10D to 8D. For types VIII and IX, i.e. the semi-simple and simple cases, this reduction is a

Kaluza–Klein reduction over \mathbb{H}^2 and S^2 , respectively (the latter of which was already noted in [2]).

The remaining non-semi-simple cases have $q_1 = 0$ and therefore (10) is independent of z^2 as well. The group manifold reduction can therefore be split up in a trivial reduction over T^2 followed by a reduction with a twist from 9D to 8D. For class A the twist is an element of the $SL(2, \mathbb{R})$ symmetry of the action, while class B also involves the \mathbb{R} trombone symmetry of the 9D field equations (which scales the 9D Lagrangian). The analogous reduction from 10D to 9D was discussed in [14].

3. Domain wall solutions

In [7], we obtained the most general half-supersymmetric domain wall solutions of the class A supergravities:

$$\begin{aligned} ds^2 &= H^{\frac{1}{12}} dx_7^2 - H^{-\frac{5}{12}} dy^2, \\ e^\varphi &= H^{\frac{1}{4}}, \quad e^\sigma = H^{-\frac{1}{2\sqrt{3}}} h_1^{\frac{\sqrt{3}}{2}}, \quad e^\phi = H^{-\frac{1}{2}} h_1^{\frac{1}{2}} h_2, \\ \chi_1 &= \chi_2 = \chi_3 = 0, \end{aligned} \quad (15)$$

where the dependence on the transverse coordinate y is governed by

$$H(y) = h_1 h_2 h_3, \quad \text{with} \quad h_m \equiv q_m y + c_m. \quad (16)$$

Here c_m are arbitrary constants whose values will affect the range of y , due to the obvious requirement $h_m > 0$. The Killing spinor satisfies the condition

$$(1 + \Gamma_{y123})\epsilon = 0, \quad (17)$$

where the indices 1, 2, 3 refer to the internal group manifold directions. Note that the dependence on the transverse coordinate y is expressed in terms of three functions h_m which are harmonic on \mathbb{R} . We define n to be the number of linearly independent harmonics h_m with $q_m \neq 0$. The maximal value of n in a specific class is then given by the number of non-zero q_m of the corresponding structure constants. We call the solution an n -tuple domain wall with $n \leq 3$. In this terminology, $n = 3$ gives a triple, $n = 2$ a double and $n = 1$ a single domain wall, while $n = 0$ is flat spacetime [7].

Upon uplifting to $D = 11$, using relation (3), we find that the n -tuple domain wall solutions become purely gravitational solutions with a metric of the form $\hat{ds}^2 = dx_7^2 - ds_4^2$, where

$$ds_4^2 = H^{-\frac{1}{2}} dy^2 + H^{\frac{1}{2}} \left(\frac{\sigma_1^2}{h_1} + \frac{\sigma_2^2}{h_2} + \frac{\sigma_3^2}{h_3} \right). \quad (18)$$

This is an extension to different Bianchi types of the generalized Eguchi–Hanson solution constructed in [15]. In general the metrics (18) are singular. There are two exceptions to this behaviour, with all q_a equal (i.e. the type IX or $SO(3)$ case) and

- three c_a equal. In this case the manifold becomes (locally) flat with $\mathbb{R} \times S^3 = \mathbb{E}^4$.
- $c_1 = c_2 = 1$ and $c_3 = 0$. In this case the manifold becomes the 4D Eguchi–Hanson manifold with $SO(3)$ isometry group and $SU(2)$ holonomy group. It is asymptotically locally Euclidean: $\mathbb{R} \times S^3/\mathbb{Z}_2 = \mathbb{R} \times SO(3)$.

These non-singular possibilities correspond to the near-horizon limit of the single- [2] and double-centred [7] Kaluza–Klein monopole, respectively.

We would like to see whether there are also supersymmetric domain wall solutions to the class B supergravities. The structure of the BPS equations requires the projector for the

Killing spinor of a 1/2 BPS domain wall solution to be the same as above. The presence of the extra term in $\delta\psi_\mu$ (12), depending on the trace of the structure constants, implies that there are no domain wall solutions with this type of Killing spinor, since the structure of Γ -matrices of this term cannot be combined with other terms. To get a solution, one is forced to put $f_{ij}{}^j = 0$, thus leading back to the class A case. This also follows from $\delta\lambda_i$, since the resulting equation is symmetric in two indices, except for a single antisymmetric term, containing $f_{ij}{}^j$. Next, we search for domain wall solutions preserving an arbitrary fraction of the supersymmetry. From the structure of the BPS equations, it is seen that only one additional kind of projector is allowed, namely

$$(1 + \Gamma_{\alpha 123})\epsilon = 0, \quad (19)$$

where $\alpha \neq y$ and spacelike. However, this again leads to $f_{ij}{}^j = 0$. We conclude that there are no domain wall solutions preserving any fraction of supersymmetry for the class B supergravities.

4. Conclusions

In this work we have constructed 11 maximal $D = 8$ gauged supergravities in terms of the Bianchi classification of three-dimensional Lie groups, which distinguishes between classes A and B. We find that this distinction carries over to a number of features of the eight-dimensional theories. Class A theories can be formulated in terms of an action, whereas the theories of class B have equations of motion that cannot be integrated to an action.

Moreover, we showed that the class A gauged supergravities allow for 1/2 BPS domain walls while there are no domain wall solutions for the class B theories that preserve any supersymmetry. The class A solutions are given by so-called n -tuple domain walls with $n \leq 3$, which can be viewed as the superposition of n domain walls. The embedding of these n -tuple domain wall solutions into M-theory leads to purely gravitational solutions: the direct product of 7D Minkowski spacetime and a 4D Euclidean geometry.

In the $SO(3)$ case, these solutions are conjectured to be related to the near-horizon limit of the double-centred Kaluza–Klein monopole by the DW/QFT correspondence. In [2], the $n = 1$ domain wall was found to correspond to the one-centre monopole, while we found in [7] that the $n = 2$ case uplifted to the near-horizon limit of the two-centre monopole solution. Both these solutions are non-singular upon uplifting whereas the $n = 3$ domain wall uplifts to a singular spacetime [15].

Acknowledgments

DR wishes to thank the organizers of the RTN workshop in Copenhagen in 2003 for the organization of the very nice workshop and the opportunity to give a presentation. TO would like to thank the CERN Theory Division for its hospitality and financial support and M M Fernández for her continuous support. This work is supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime, in which the University of Groningen is associated with the University of Utrecht. The work of UG is part of the research programme of the 'Stichting voor Fundamenteel Onderzoek der Materie' (FOM). The work of TO is partially supported by the Spanish grant BFM2003-01090.

References

- [1] Salam A and Sezgin E 1985 $D = 8$ supergravity *Nucl. Phys. B* **258** 284
- [2] Boonstra H J, Skenderis K and Townsend P K 1999 The domain wall/QFT correspondence *J. High Energy Phys.* JHEP01(1999)003 (*Preprint hep-th/9807137*)
- [3] Edelstein J D and Núñez C 2001 D6 branes and M-theory geometrical transitions from gauged supergravity *J. High Energy Phys.* JHEP04(2001)028 (*Preprint hep-th/0103167*)
- [4] Hernández R 2001 Branes wrapped on coassociative cycles *Phys. Lett. B* **521** 371 (*Preprint hep-th/0106055*)
- [5] Gomis J and Mateos T 2002 D6 branes wrapping Kähler four-cycles *Phys. Lett. B* **524** 170 (*Preprint hep-th/0108080*)
- [6] Hernández R and Sfetsos K 2002 An eight-dimensional approach to $G(2)$ manifolds *Phys. Lett. B* **536** 294 (*Preprint hep-th/0202135*)
- [7] Alonso-Alberca N, Bergshoeff E, Gran U, Linares R, Roest D and Ortín T 2003 Domain walls of $D = 8$ gauged supergravities and their $D = 11$ origin *J. High Energy Phys.* JHEP06(2003)038 (*Preprint hep-th/0303113*)
- [8] Bergshoeff E *et al* 2003 The Bianchi classification of maximal $D = 8$ gauged supergravities *Class. Quantum Grav.* **20** 3997–4014 (*Preprint hep-th/0306179*)
- [9] Dewitt B S 1964 *Relativity, Groups and Topology (Les Houches 1963)* (New York: Gordon and Breach)
- [10] Scherk J and Schwarz J H 1979 How to get masses from extra dimensions *Nucl. Phys. B* **153** 61
- [11] Bianchi L 1898 Sugli spazi a tre dimensioni che ammettono un gruppo continuo di movimenti *Mem. Mat. Fis. Soc. Ital. Sci. Serie Terza* **11** 267 See www34.homepage.villanova.edu/robert.jantzen/bianchi/bianchi.html for details of an English translation
- [12] Cvetič M, Gibbons G W, Lu H and Pope C N 2003 Consistent group and coset reductions of the bosonic string *Preprint hep-th/0306043*
- [13] Pons J M and Shepley L C 1998 Dimensional reduction and gauge group reduction in Bianchi-type cosmology *Phys. Rev. D* **58** 024001 (*Preprint gr-qc/9805030*)
- [14] Bergshoeff E, de Wit T, Gran U, Linares R and Roest D 2002 (Non-)Abelian gauged supergravities in nine dimensions *J. High Energy Phys.* JHEP10(2002)061 (*Preprint hep-th/0209205*)
- [15] Belinskii V, Gibbons G, Page D and Pope C 1978 Asymptotically Euclidean Bianchi IX metrics in quantum gravity *Phys. Lett. B* **76** 433